Computational Spacetime: A Framework for Emergent Geometry and Temporal Direction from Tractable Information Measures

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Abstract

We present a computational framework that derives spacetime geometry and temporal direction from tractable information-theoretic principles. Our approach resolves the uncomputability crisis in "It from Bit" theories by introducing two genuinely computable metrics: Bounded Causal Algorithmic Depth (Γ_T) for spatial structure and Practical Compression Asymmetry (Λ_C) for temporal direction. We demonstrate that these metrics, while computationally intensive, remain within the bounds of physical computability. From minimal postulates, we derive conditions that constrain spacetime transformations and provide a computational foundation for understanding temporal irreversibility. The framework generates four testable predictions, establishing a research program that connects fundamental physics to computational complexity theory.

Keywords: computational physics, emergent spacetime, algorithmic information theory, arrow of time, computable complexity.

1 Introduction

Information-theoretic approaches to fundamental physics have shown promise in deriving spacetime and physical laws from computational principles [1]. However, these "It from Bit" theories typically invoke Kolmogorov Complexity $K(x)$, which requires solving the halting problem and is therefore uncomputable [2, 3]. This creates what we term the "uncomputability crisis": physical theories cannot be grounded in quantities that no physical process can evaluate.

We present a framework that replaces uncomputable measures with tractable alternatives while maintaining the essential insights of information-theoretic physics. Our approach recognizes that physical computability requires not just theoretical computability, but practical tractability within cosmic timescales, a constraint inspired by ultimate physical limits on computation [4].

2 Foundations: Tractable Complexity Measures

2.1 Physical Computability Constraint

Postulate 1 (Bounded Computational Realism)**.** *Physical laws must be expressible using functions that can be computed within the available computational resources of the observable universe, accounting for fundamental physical limits on computation.*

This postulate is stronger than mere theoretical computability—it requires practical tractability.

2.2 Bounded Causal Algorithmic Depth

Definition 2.1 (Causal Graph)**.** *The universe is modeled as a discrete structure $G = (E, C)$, where E represents fundamental causal events and C represents direct causal influences.*

Definition 2.2 (Bounded Causal Algorithmic Depth)**.** *For an event $e \in E$, its Bounded Causal Algorithmic Depth is defined as:*

$$\Gamma_T(e) := \min\{runtime(P) \mid U(P, G_0) \text{ generates } e \text{ in time } \leq T\} \tag{1}$$

where T is a cosmic time bound (e.g., the age of the universe), U is a fixed universal Turing machine, and G_0 is a primordial graph state. For any program that fails to generate e within time T, we consider its runtime to be T.

Remark 2.1. This metric is computable by an exhaustive search over all programs up to a length whose runtime can be simulated within time T. It is tractable in principle for events of bounded complexity, in stark contrast to uncomputable measures like standard logical depth [7].

2.3 Practical Compression Asymmetry

To avoid the uncomputability of Kolmogorov Complexity, we define an asymmetry based on practical compressors.

Definition 2.3 (Practical Compression Asymmetry)**.** *For a sequence of system states $S_1 \to S_2 \to \cdots \to S_n$, using a fixed, practical compressor $\mathcal{C}$ (e.g., Lempel-Ziv [5]):*

$$\Lambda_C(S_1 \to S_n) := \ln\left(\frac{|\mathcal{C}(S_n, S_{n-1}, \ldots, S_1)|}{|\mathcal{C}(S_1, S_2, \ldots, S_n)|}\right) \tag{2}$$

where $|\mathcal{C}(\cdot)|$ denotes the length of the compressed output.

Remark 2.2. $\Lambda_C > 0$ indicates that the forward temporal evolution produces a more algorithmically compressible description than the time-reversed sequence. The choice of compressor $\mathcal{C}$ is a key component of the model, which we argue should reflect the information-processing capabilities inherent in physical systems.

3 Coordinate Construction and Physical Principles

3.1 Computational Coordinate System

Definition 3.1 (Computational Radar Coordinates). *For an observer at event e_i measuring an event e via a return signal at event e_j, the spacetime coordinates are constructed as:*

$$t_O(e) = \frac{\Gamma_T(e_j) + \Gamma_T(e_i)}{2R_{max}} \tag{3}$$

$$r_O(e) = \frac{|\Gamma_T(e_j) - \Gamma_T(e_i)|}{2R_{max}} \cdot \ell_{comp} \tag{4}$$

where R_{max} is the maximum possible computational depth within the cosmic time T, and ℓ_{comp} is a fundamental length scale relating computation to geometry. This construction is justified by interpreting computational depth as a proxy for causal distance, analogous to standard radar coordinates. The factor R_{max} normalizes the quantities.

3.2 Event Density Principle

Postulate 2 (Computational Event Density). *The density of fundamental events is proportional to the local computational intensity:*

$$\rho(x) = \rho_0 \left(1 + \alpha \frac{\partial \Gamma_T}{\partial x^\mu} \cdot n^\mu + \beta \Lambda_C\right) \tag{5}$$

where n^μ is a normalized direction vector, α and β are dimensionless coupling constants, and ρ_0 is a baseline event density. This postulate is motivated by the idea that regions of intense computation (high depth gradients or temporal asymmetry) should manifest as denser regions of physical reality.

4 Derivation of Physical Constraints

4.1 Spacetime Transformation Constraints

Lemma 4.1 (Event Conservation). *The total number of events N in any spacetime region $\mathcal{V}$ is an observer-independent scalar. Thus, $N = \int_{\mathcal{V}} \rho(x)\, d^4x =$ invariant.*

Theorem 4.1 (Computational Symmetry Constraints). *For the total event count to remain invariant under a coordinate transformation $x \to x' = \Lambda x$, the transformation matrix $\Lambda^\mu_{\ \nu}$ must satisfy:*

$$|\det(\Lambda)| \cdot J[\Gamma_T, \Lambda_C] = 1 \tag{6}$$

where $J[\Gamma_T, \Lambda_C]$ is the Jacobian accounting for the transformation of the computational metrics.

Proof. See Appendix A.1 for the detailed derivation. □

Corollary 4.1. *For transformations that preserve the computational structure of spacetime (i.e., $J = 1$), we recover the familiar constraint $|\det(\Lambda)| = 1$, which is a defining property of the proper orthochronous Lorentz group.*

4.2 Emergence of a Fundamental Speed Limit

Theorem 4.2 (Computational Speed Limit)**.** *The maximum rate of computational depth generation imposes a fundamental speed limit on the propagation of causal influence,* $c = \ell_{comp}$*.*

Proof. See Appendix A.2 for the detailed derivation. □

4.3 Temporal Direction

Theorem 4.3 (Emergence of Temporal Direction)**.** *For any macroscopic system evolving from a simple, low-entropy state toward thermodynamic equilibrium, we expect* $\Lambda_C > 0$*.*

Proof. See Appendix A.3 for the detailed derivation. □

5 Testable Predictions

Our framework makes four specific, operationally defined predictions.

1. **Computational Dispersion in Quantum Systems:** Quantum states that require higher computational depth to prepare will exhibit measurable deviations in their evolution rates.
$$\Delta t_{\text{evolution}} \propto \frac{\Gamma_T(\psi_{\text{initial}})}{R_{\max}} \tag{7}$$
Test Protocol: Prepare quantum states of varying computational complexity (e.g., using quantum circuits of different depths, where depth is a proxy for Γ_T) and measure their evolution timescales using standard quantum process tomography.

2. **Universal Compression Asymmetry in Irreversible Processes:** All irreversible physical processes will show $\Lambda_C > 0$ when analyzed using standard compression algorithms.
$$\Lambda_C(\text{process}) > \epsilon > 0 \quad \text{(for e.g., Lempel-Ziv)} \tag{8}$$
Test Protocol: Record high-resolution data from irreversible processes (e.g., chemical reactions, fluid turbulence, radioactive decay) and apply compression analysis to the forward and time-reversed data streams.

3. **Gravitational Depth Gradients:** Near massive objects, the gradient of computational depth should correlate with the strength of the gravitational field.
$$\frac{d\Gamma_T}{dr} \propto \frac{GM}{r^2c^2} \tag{9}$$
Test Protocol: Use astronomical observations to search for correlations between complexity measures of astrophysical processes (e.g., pulsar timing variations) and their gravitational environment.

4. **Fundamental Information Processing Bounds:** Physical systems should exhibit maximum information processing rates that scale with their computational depth.
$$\frac{dI_{\text{processed}}}{dt} \leq f(\Gamma_T(\text{system})) \tag{10}$$

Test Protocol: Measure information processing rates in diverse physical systems (biological, technological, quantum) and test for the predicted scaling relationship against a suitable proxy for system depth.

6 Connections to Established Physics

Thermodynamics Our compression asymmetry Λ_C provides a computational basis for the second law. It can be directly related to entropy production:

$$\frac{dS}{dt} \approx k_B \cdot \frac{d\Lambda_C}{dt} \cdot \mathcal{N} \tag{11}$$

where $\mathcal{N}$ is a normalization factor related to the system's degrees of freedom.

Relativistic Constraints While we do not derive the full Lorentz group, our framework establishes a fundamental principle—event conservation—that constrains spacetime transformations, suggesting a path toward deriving relativistic symmetries from computational invariance.

Quantum Mechanics The concept of bounded computational depth suggests a natural, finite information content for any region of spacetime, potentially connecting to holographic principles and theories of emergent quantum mechanics from discrete computational processes [6].

7 Limitations and Future Directions

This framework, while promising, has several key limitations that define our future research directions.

Computational Cost: Exact calculation of Γ_T remains computationally prohibitive for complex systems, necessitating reliance on approximation schemes like hierarchical memoization, Monte Carlo methods, or machine learning proxies.

Model Dependence: The framework depends on the choice of universal machine U and compressor $\mathcal{C}$. Justifying a canonical choice for these components, or proving universality class independence, is a major challenge.

Symmetry Group: We have derived constraints on symmetries but not yet the full Lorentz or Poincaré group from first principles.

Quantum Extension: The present framework is primarily classical. A full quantum-mechanical extension, defining quantum algorithmic depth and quantum compression asymmetry, is a critical next step.

8 Conclusion

We have presented a computational framework for spacetime that avoids the uncomputability crisis through the use of bounded, tractable complexity measures. Our approach generates specific, testable predictions while remaining grounded in practical computational principles. The framework suggests that the structure of spacetime and the

arrow of time may be emergent reflections of the computational architecture underlying physical reality. While significant work remains, this approach establishes a viable research program connecting fundamental physics to computational complexity theory, respecting the crucial constraint that physical theories must be implementable by physical processes.

References

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A Detailed Mathematical Derivations

A.1 Proof of Theorem 4.1 (Computational Symmetry Constraints)

Proof. We begin with the principle of Event Conservation (Lemma 4.1), which states that the total number of fundamental events N in a spacetime volume $\mathcal{V}$ is an invariant scalar.

$$N = \int_{\mathcal{V}} \rho(x)\, d^4x = \int_{\mathcal{V}'} \rho'(x')\, d^4x' \tag{12}$$

where the primed frame is related to the unprimed frame by the transformation $x' = \Lambda x$.

Using the standard rule for change of integration variables, the volume element transforms as $d^4x' = |\det(\Lambda)| d^4x$. Substituting this into the integral equality gives:

$$\int_{\mathcal{V}} \rho(x)\, d^4x = \int_{\mathcal{V}} \rho'(\Lambda x) |\det(\Lambda)|\, d^4x \tag{13}$$

For this equality to hold for any arbitrary volume $\mathcal{V}$, the integrands must be equal:

$$\rho(x) = \rho'(\Lambda x) |\det(\Lambda)| \tag{14}$$

The density function $\rho(x)$ is defined in Postulate 2. Let us denote the computational part as $S(x) = 1 + \alpha(\partial\Gamma_T/\partial x^\mu)n^\mu + \beta\Lambda_C$. Then $\rho(x) = \rho_0 S(x)$. The density function in the new frame, $\rho'(x')$, has the same form but with transformed computational quantities: $\rho'(x') = \rho_0 S'(x')$.

The transformation of the computational structure can be captured by a Jacobian factor J, which we define as the ratio of the computational terms:

$$J \equiv \frac{S'(x')}{S(x)} \tag{15}$$

Substituting $\rho(x) = \rho_0 S(x)$ and $\rho'(x') = \rho_0 S'(x') = \rho_0 J S(x)$ into our integrand equality (14):

$$\rho_0 S(x) = (\rho_0 J S(x)) |\det(\Lambda)| \tag{16}$$

Assuming $S(x) \neq 0$ and $\rho_0 \neq 0$, we can divide through by these terms, yielding the final constraint:

$$1 = J |\det(\Lambda)| \tag{17}$$

This demonstrates that any valid coordinate transformation must either preserve the computational structure ($J = 1$) and have unit determinant, or have its volume-changing effect precisely canceled by a corresponding change in the local computational density. □

A.2 Proof of Theorem 4.2 (Computational Speed Limit)

Proof. The proof relies on the coordinate definitions (3) and (4) and the fundamental physical constraint that computational depth cannot be generated infinitely fast.

Let's consider the change in the time and space coordinates, dt and dr, for an infinitesimal propagation step. From the definitions:

$$dt = \frac{d\Gamma_T(e_j) + d\Gamma_T(e_i)}{2R_{\max}} \tag{18}$$

$$dr = \frac{|d\Gamma_T(e_j) - d\Gamma_T(e_i)|}{2R_{\max}} \cdot \ell_{\text{comp}} \tag{19}$$

Let $R_i = d\Gamma_T(e_i)/d\tau$ and $R_j = d\Gamma_T(e_j)/d\tau$ be the rates of depth generation for the emission and return events with respect to some fundamental clock tick process $d\tau$. By definition, these rates are bounded: $0 \leq R_i, R_j \leq R_{\max}$.

The instantaneous velocity v is the ratio dr/dt:

$$v = \frac{dr}{dt} = \frac{\frac{|R_j - R_i| d\tau}{2R_{\max}} \cdot \ell_{\text{comp}}}{\frac{(R_j + R_i)d\tau}{2R_{\max}}} = \ell_{\text{comp}} \frac{|R_j - R_i|}{R_j + R_i} \tag{20}$$

To find the maximum possible velocity, we must maximize the dimensionless ratio $f(R_i, R_j) = |R_j - R_i|/(R_j + R_i)$ subject to the constraints $0 \leq R_i, R_j \leq R_{\max}$.

The function f is maximized when the difference between R_i and R_j is maximized. This occurs at the boundaries of the domain. Let's set one rate to its maximum, $R_j = R_{\max}$, and the other to its minimum, $R_i \to 0$.

$$v_{\max} = \lim_{R_i \to 0} \ell_{\text{comp}} \frac{R_{\max} - R_i}{R_{\max} + R_i} = \ell_{\text{comp}} \frac{R_{\max}}{R_{\max}} = \ell_{\text{comp}} \tag{21}$$

This corresponds to a signal that is generated with minimal computational effort (e.g., a simple photon emission, $R_i \approx 0$) and propagates in a way that generates depth at the maximum possible rate ($R_j = R_{\max}$).

Thus, the framework imposes a fundamental speed limit on the propagation of information, $c \equiv \ell_{\text{comp}}$, which emerges directly from the finite rate of computational depth generation in the universe. □

A.3 Proof of Theorem 4.3 (Emergence of Temporal Direction)

Proof. Consider a macroscopic system undergoing an irreversible process, evolving from a simple, low-entropy initial state S_1 to a complex, high-entropy equilibrium state S_n.

1. **Forward Process ($S_1 \to S_n$):** The state S_1 is simple (e.g., all gas in one corner of a box). A short computer program can describe S_1. The evolution $S_1 \to S_n$ is governed by simple, local physical laws (e.g., the laws of gas dynamics). Therefore, the entire forward sequence can be described compactly by a program that specifies S_1 and the laws of evolution. A good compressor $\mathcal{C}$ will find this compact description.

$$|\mathcal{C}(S_1, \ldots, S_n)| \approx |\mathcal{C}(S_1)| + |\mathcal{C}(\text{Physical Laws})| \tag{22}$$

2. **Reverse Process ($S_n \to S_1$):** The state S_n is a generic, high-entropy equilibrium state. It is algorithmically random and incompressible, so $|\mathcal{C}(S_n)|$ is large. To reverse the process and evolve from S_n to the specific low-entropy state S_1, one must not only know the (time-reversed) physical laws but also specify the astronomically unlikely set of microstate conditions (e.g., the precise velocities of all particles) that would conspire to produce S_1. This requires a vast amount of additional information.

$$|\mathcal{C}(S_n, \ldots, S_1)| \approx |\mathcal{C}(S_n)| + |\mathcal{C}(\text{Reversed Laws})| + |\mathcal{C}(\text{Fine-tuning Info})| \tag{23}$$

The "fine-tuning information" term is enormous, reflecting the fact that almost all microstates corresponding to S_n will not evolve back to S_1.

3. **Comparison:** Because the fine-tuning information is required for the reverse path but not the forward path, and because the initial state S_1 is far more compressible than the final state S_n, we have:

$$|\mathcal{C}(S_n, \ldots, S_1)| \gg |\mathcal{C}(S_1, \ldots, S_n)| \tag{24}$$

Therefore, the Practical Compression Asymmetry $\Lambda_C = \ln\left(\frac{|\mathcal{C}(\text{reverse})|}{|\mathcal{C}(\text{forward})|}\right)$ will be significantly greater than zero for any such process. This provides a computational basis for the thermodynamic arrow of time. □